The structure and origin of confined Holmboe waves

A. Lefauve*, P. F. Linden*, J. L. Partridge* Qi Zhou†, S. B. Dalziel* and C. P. Caulfield*‡

Finite-amplitude manifestations of stratified shear flow instabilities and their spatio-temporal coherent structures are believed to play an important role in turbulent geophysical flows. Such shear flows commonly have layers separated by sharp density interfaces, and are therefore susceptible to the so-called Holmboe instability, and its finite-amplitude manifestation, the Holmboe wave. In this paper, we describe and elucidate the origin of an apparently previously unreported long-lived coherent structure in a laboratory stratified shear flow generated by exchange flow through an inclined square duct connecting two reservoirs filled with fluids at different densities. Using a novel measurement technique allowing for time-resolved, near-instantaneous measurements of the three-component velocity and density fields simultaneously over a three-dimensional volume, we describe the three-dimensional geometry and spatio-temporal dynamics of this structure. We identify it as a finite-amplitude, nonlinear, asymmetric *confined Holmboe wave* (CHW), and highlight the importance of its spanwise (lateral) confinement by the duct boundaries. We pay particular attention to the spanwise vorticity, which exhibits a travelling, near-periodic structure of sheared, distorted, prolate spheroids with a wide 'body' and a narrower 'head'. Using temporal linear stability analysis on the two-dimensional streamwise-averaged experimental flow, we solve for three-dimensional perturbations having two-dimensional, cross-sectionally confined eigenfunctions and a streamwise normal mode. We show that the dispersion relation and the three-dimensional spatial structure of the predicted fastest growing *confined Holmboe instability* (CHI) are in good agreement with those of the observed CHW. We also compare those results with a classical linear analysis of two-dimensional perturbations (i.e. with no spanwise dependence) on a one-dimensional base flow. We conclude that the lateral confinement is an important ingredient of the CHI, which we argue is at the origin of the CHW, with implications for many inherently confined geophysical flows such as in valleys, estuaries, straits or deep ocean trenches. Our results suggest that the CHW is an example of an experimentally observed, inherently nonlinear, robust, long-lived coherent structure which has developed from a linear instability. We conjecture that the CHW is a promising candidate for a class of exact coherent states underpinning the dynamics of more disordered, yet continually forced stratified shear flows.

*DAMTP, University of Cambridge, Cambridge, CB3 0WA, UK.
†Department of Civil Engineering, University of Calgary, Calgary, Alberta T2N 1N4, Canada
‡BP Institute, University of Cambridge, Cambridge CB3 0EZ, UK