Variational methods for finding periodic orbits in turbulence

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Unstable periodic orbits are believed to underpin the dynamics of turbulence, but by their nature are hard to find computationally. Historically the most successful methods have been based on shooting, i.e. optimising an initial condition so that when its trajectory is followed, it reintersects with the initial condition. These shooting methods typically require very good initial guesses for convergence to be achieved. Alternative methods based on deforming a closed loop until it is everywhere tangent to the dynamical system have shown promise in converging less accurate initial guesses[1,2], but several complications exist when applying these to fluid dynamics. We present a family of methods to converge unstable periodic orbits for the incompressible Navier-Stokes equations, based on variations of an integral objective functional, and using traditional gradient-based optimisation strategies. Different approaches for handling the incompressibility condition are considered. The variational methods are applied to the specific case of periodic, two-dimensional Kolmogorov flow and compared against existing Newton iteration-based shooting methods. While computationally slow, our methods converge from very inaccurate initial guesses, and can be combined with shooting methods to give an efficient strategy for converging many periodic orbits.

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[1] Lan & Cvitanović *Phys. Rev. E* **69**, 016217 (2004).
[2] Lasagna *SIAM J. Appl. Dyn. Syst.* **17**, 547 (2018).

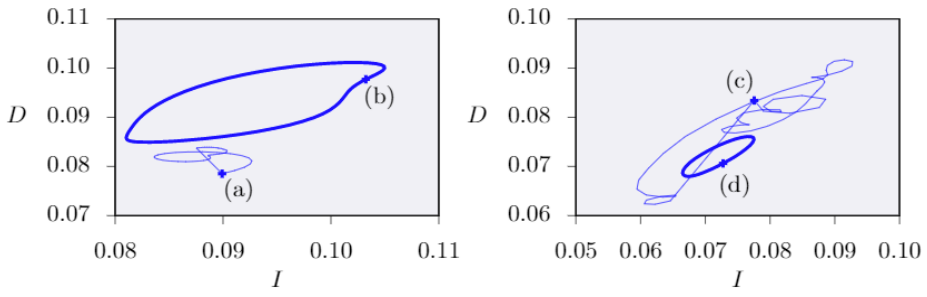

Figure 1: Energy input-dissipation plots of initial guesses (thin lines) and converged periodic orbits (thick lines) for two different examples. Note the ability to converge from very poor initial guesses.