A variational approach to boundary effects in higher-order homogenization

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From an engineering point of view, it is convenient to describe composite materials using homogeneous effective properties. When the microstructure is periodic, asymptotic homogenization is particularly well suited for this aim. Classical homogenization corresponds to the dominant order model and yields an effective standard Cauchy medium. At next orders, we can derive additional corrections that depend on the successive strain gradients. These corrections are typically of interest to capture size-effects appearing for microstructures with contrasted stiffness properties.

However, these higher-order models present one major limitation: the corrections produced by homogenization can handle size-effects that occur in the bulk region, but are not suited to the analysis of the boundaries. In fact, they miss significant boundary effects which can degrade the quality of the predictions. Besides, these boundary effects contribute significantly to the total energy of the system. By ignoring these contributions, one can break the variational structure of the problem. Indeed, most effective energies obtained by neglecting such contributions are unexploitable, as they generate ill-posed boundary-value problems that cannot even be solved order by order. To handle these limitations, we elaborate a new homogenization procedure that includes boundary effects. By contrast with usual approaches, in our procedure the homogenization is carried at the energy level, rather than on the strong form of the equilibrium.

As an example, we consider a 2D laminate that is invariant in the direction perpendicular to the fibers. The resulting effective energy contains a bulk term that captures strain-gradient effects, plus a boundary term that accounts for the energy generated by the boundary effects. The effective stiffness on boundary is computed numerically, by combining solutions of elementary 2D elastic problems formulated on a semi-infinite strip. We demonstrate that, by contrast with usual asymptotic homogenization, this effective higher-order model is able to capture size-effects occurring in the interior domain, as well as near the boundaries. Besides, we show that the boundary terms appearing in the effective energy satisfy compatibility conditions, which guarantee that the boundary-value problem can be solved order by order.