Control-based continuation of an externally excited MEMS self-oscillator

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Summary. We apply control-based continuation (CBC) to a periodically forced active micro-cantilever with integrated sensing and actuation elements. Previous work with an experimental test rig has shown that this micro-electromechanical system (MEMS) is described well by a set of ordinary differential equations. We use this mathematical model to demonstrate that CBC with non-invasive position control is able to follow synchronised periodic solutions through stability changes at fold points.

Background

Externally forced, weakly nonlinear systems and, particularly, self-oscillating structures are known to exhibit synchronisation [1]. This type of entrained dynamics generally occur when the driving oscillations quite closely match the intrinsic, self-sustained oscillations of the system. Non-invasive control techniques have been applied to expand the parameter region with successful synchronisation [2]. Control-based continuation (CBC) is an an experimental methodology used to explore the open-loop dynamics of a nonlinear system, where co-existing solutions are present. It can explore both stable and unstable solutions by tracking continuously through fold bifurcations. CBC has been applied to mechanical structures on the macro-scale [3, 4, 5]. However, its practicality for structures on the micro-scale has not yet been investigated. We consider here an active micro-electromechanical systems (MEMS), which has been demonstrated to feature a Hopf bifurcation to self-oscillations [6, 7]. We extend this previous work with a feasibility study that demonstrates that CBC is able to track the associated synchronised response under external excitation, irrespective of its stability.

Method and Model Description

Specifically, we consider a nonlinear, actively operated micro-cantilever with integrated piezo-resistive sensor and thermal actuator (Fig. 1c). Prior work with an experimental test rig designed to investigate the dynamics of MEMS devices characterised properties of the dynamics with a focus on the first vibration mode. The dynamics of analogue signal conditioning of electronic signals, controller mechanism and filters (Fig. 1a,b) are presented in [6].

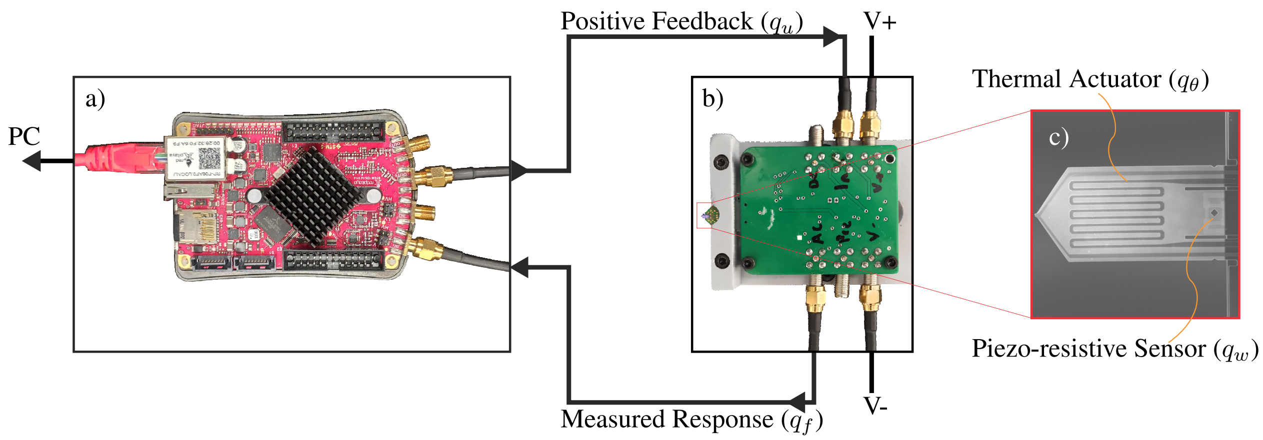

Figure 1: The experimental setup described by the MEMS mathematical model (1). a) RedPitaya STEMLab125-14; b) custom signal conditioning and amplifier circuit; and c) MEMS cantilever (taken with JEOL JSTM-IT300).

In this way, we showed that the dynamics of this active MEMS cantilever are described well by the mathematical model

$$\ddot{q}_w + \delta\dot{q}_w + q_w = \alpha q_\theta + \kappa_{ext} \quad (1a) \qquad \dot{q}_u + \omega_L q_u = \omega_L \tanh(a q_f + b) \quad (1c)$$

$$\dot{q}_\theta + \beta q_\theta = \gamma q_u^2 \quad (1b) \qquad \dot{q}_f + \omega_H q_f = \dot{q}_w \quad (1d)$$

for the mechanical deflection q_w, the thermal response q_θ, the positive feedback q_u, and the filtered signal q_f. Here the parameters α, β, γ and δ describe physical cantilever parameters, and the coefficients ω_L and ω_H refer to the filter cut-off frequencies [6]. A piezo-actuator (not shown in Fig. 1) supplies the external excitation κ_{ext} in (1a). Positive feedback is captured in (1c), as implemented by the RedPitaya STEMLab125-14 (Fig 1a). The saturation function $\tanh(\cdot)$ simulates the voltage limits, and Joule-heating physics in q_u^2 provides mechanical and thermal coupling.
The framework of CBC has been studied quite extensively in other works [4, 8, 5, 9]. The basic idea is that a zero-finding problem can be formulated by requiring that the controlled system mimics that of the open-loop behaviour. In this study,

we superimpose the control signal to the forcing signal by representing the reference trajectory r and response q_f as truncated Fourier series [5, 9]. Through fixed-point iteration, a suitable reference trajectory can be found, eliminating the need to consider higher-order harmonics. The reference trajectory acts as an arclength-like variable, in which experimental continuation can be performed. We choose as suitable control law the proportional, position-only controller:

$$\kappa_{ext} = K_p \left(r - q_f \right), \tag{2}$$

where K_p is a user-defined proportional gain. Whilst other applications of CBC require a proportional-plus derivative (PD) controller, the proportional-only controller (2) is sufficient for achieving local stabilisation for the MEMS system we study. The feedback controller only requires the state q_f, thereby eliminating computation time on derivative estimation.

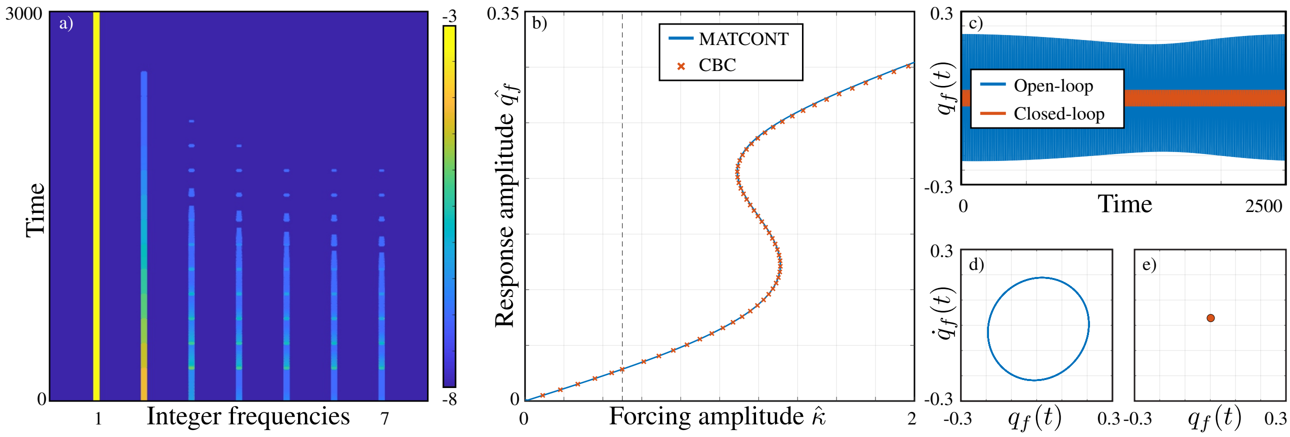

Figure 2: a) Spectrogram of the controller effort for integer frequencies (amplitude log-scale); b) S-shaped curve of synchronised responses; c) Timeseries response; d) Poincaré section of the unsynchronised and e) synchronised response.

Results and Conclusions

We demonstrate the feasibility of applying CBC to track synchronised solutions for our MEMS device represented by the dynamics (1). Figure 2a depicts the spectrogram of the controller effort (2), showing that the fixed-point iterations within the CBC algorithm ensure that the steady-state response becomes harmonic up to an acceptable tolerance. Updating and recording the reference trajectory during the (pseudo-arclength) continuation steps of CBC results in the S-shaped solution branch (shown in Fig. 2b), represented here by forcing amplitude $\hat{\kappa}$ and response amplitude $\hat{q}_f$ of the first harmonic. It is in good agreement with the open-loop dynamics, as computed with the numerical continuation package MatCont [10] directly from (1). Figure 2c shows the timeseries response of both open and closed-loop solutions. The open-loop response displays quasiperiodic behavior, whereas the CBC algorithm stabilises and tracks the unstable, synchronised response. A Poincaré section of the open loop (Fig. 2d) and closed loop (Fig. 2e) verifies this.
Following this demonstration in principle of CBC for a MEMS device, the next step is to implement it for the tracking of synchronised solutions of the actual micro-cantilever in the experiment. A specific challenge here is to achieve detection and actuation on the very fast time scales of micro-structures. We expect that CBC-enabled further insights into the rich dynamics of active MEMS cantilevers under various excitation scenarios may contribute to their increased sensitivity and performance in applications.

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